On

Damping of MAP Spacecraft

By

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Outline

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a) Basic Mechanics of CLD

b) Forms of G(w) and h(w) for VEM (ViscoElastic Material) Layer

c) Examples of Typical Material Data for VEM

II) Techniques for Representing CLD Using the Finite Element Method (FEM)

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b) Beam Elements

III) Methods of Calculation

a) Direct Method

b) Modal Method

i) Correction Factor

ii) Iterative Technique

c) Dynamic Loading Considerations

IV) Loss Factor

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a) Damping of Bolted Joints

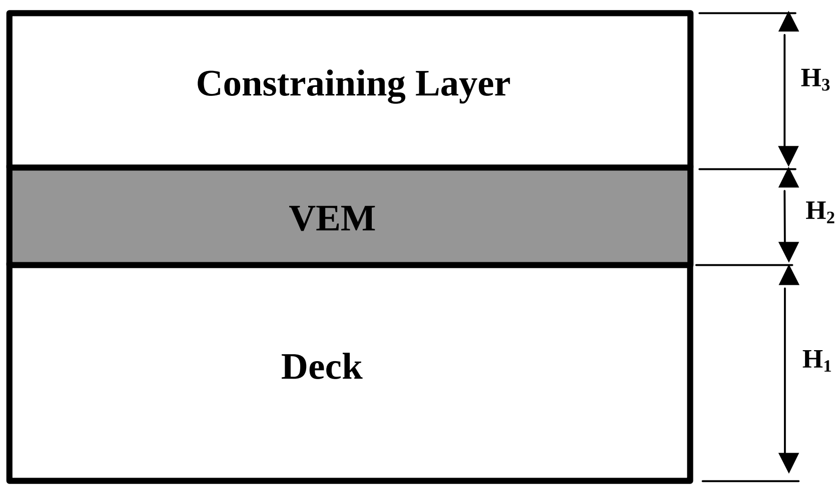
b) Struts

c) Composite/Honeycomb Panels

VII) Effects of Inertia Terms

I) Discussion of Constrained Layer Damping (CLD)

a) Basic mechanics of CLD



- **Purpose: Reduction of vibration response through energy dissipation**
- **During flexural vibration, the VEM core is constrained to shear.**
- **The shear in the core causes energy to be dissipated and the motion to be damped.**
- **Maximum damping: symmetric structure**
- **Overall loss factor h < VEM's core loss factor h_2 , where**

$$G_2^* = G_2(1 + i h_2)$$

is the VEM's complex shear modulus.

b) Form of G(*f*) and h(*f*) for VEM layer

Generally,

$$\mathbf{a}f^{\,a} + \mathbf{b}f^{\,b}$$

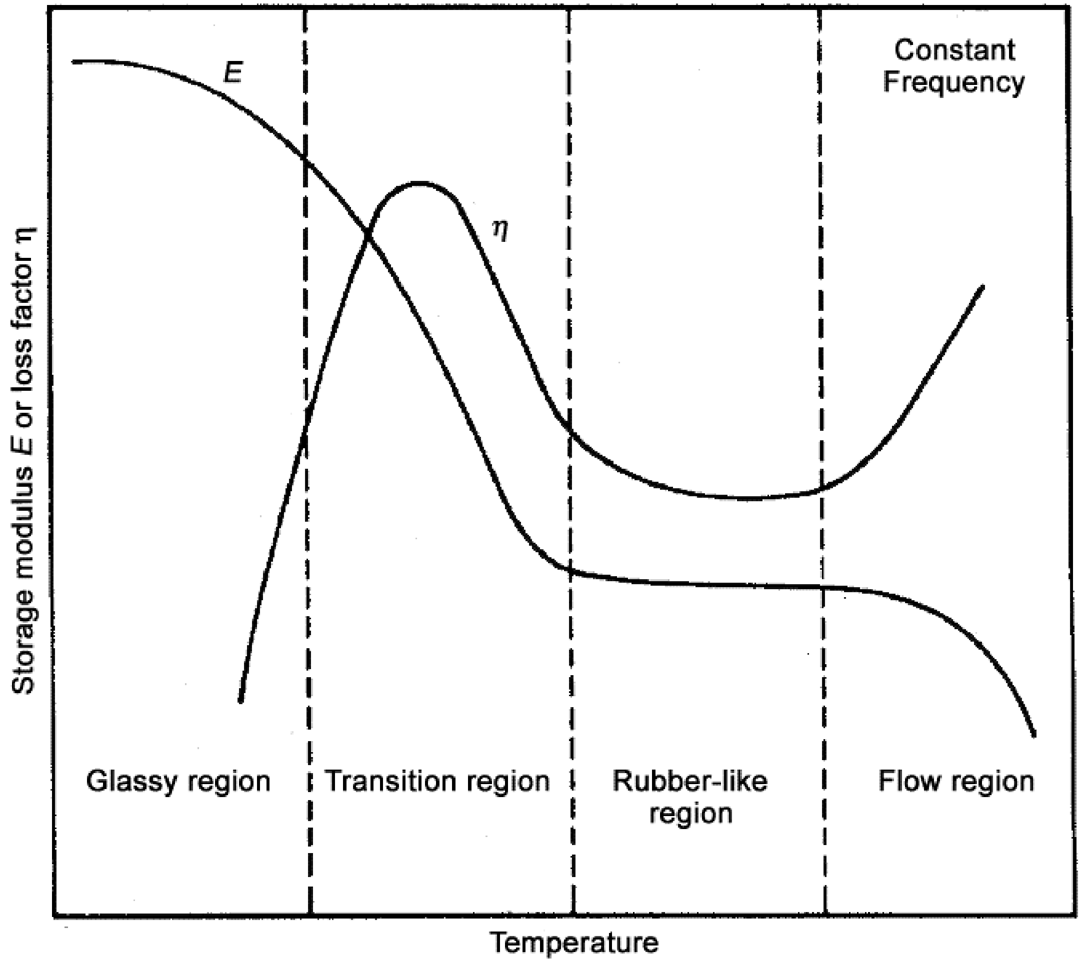
c) Example of typical material data for VEM

<u>Polymeric Materials</u>:

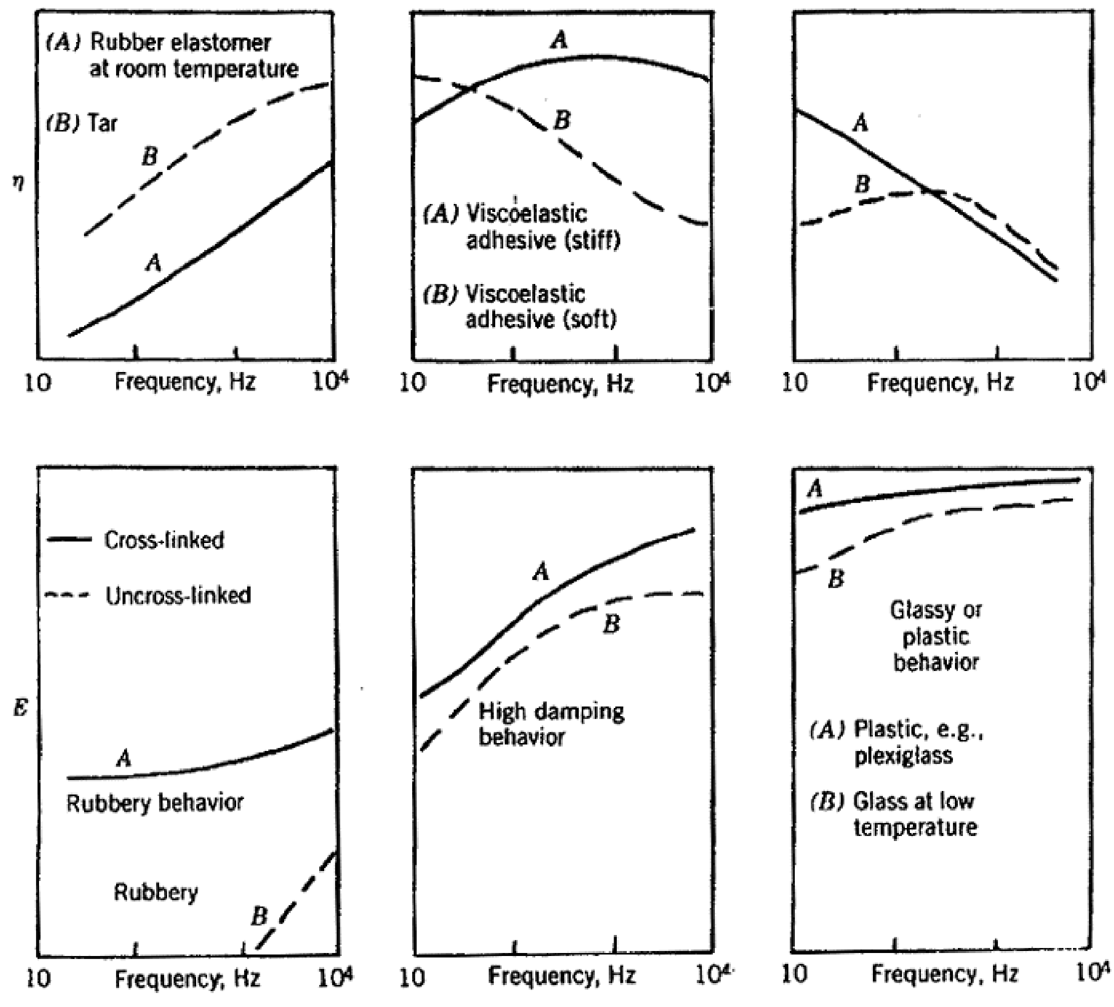
- **Long molecular chains**
- **Strong joints among carbon Atoms**
- **Damping arises from relaxation and recovery of the polymer network after it has been deformed.**

<u>Glassy Materials</u>

- **Characterized by short-term order and long –term disorder**
- **Damping arises from relaxation processes after deformation of the glass, recovery due to certain conditions of thermodynamic equilibrium.**

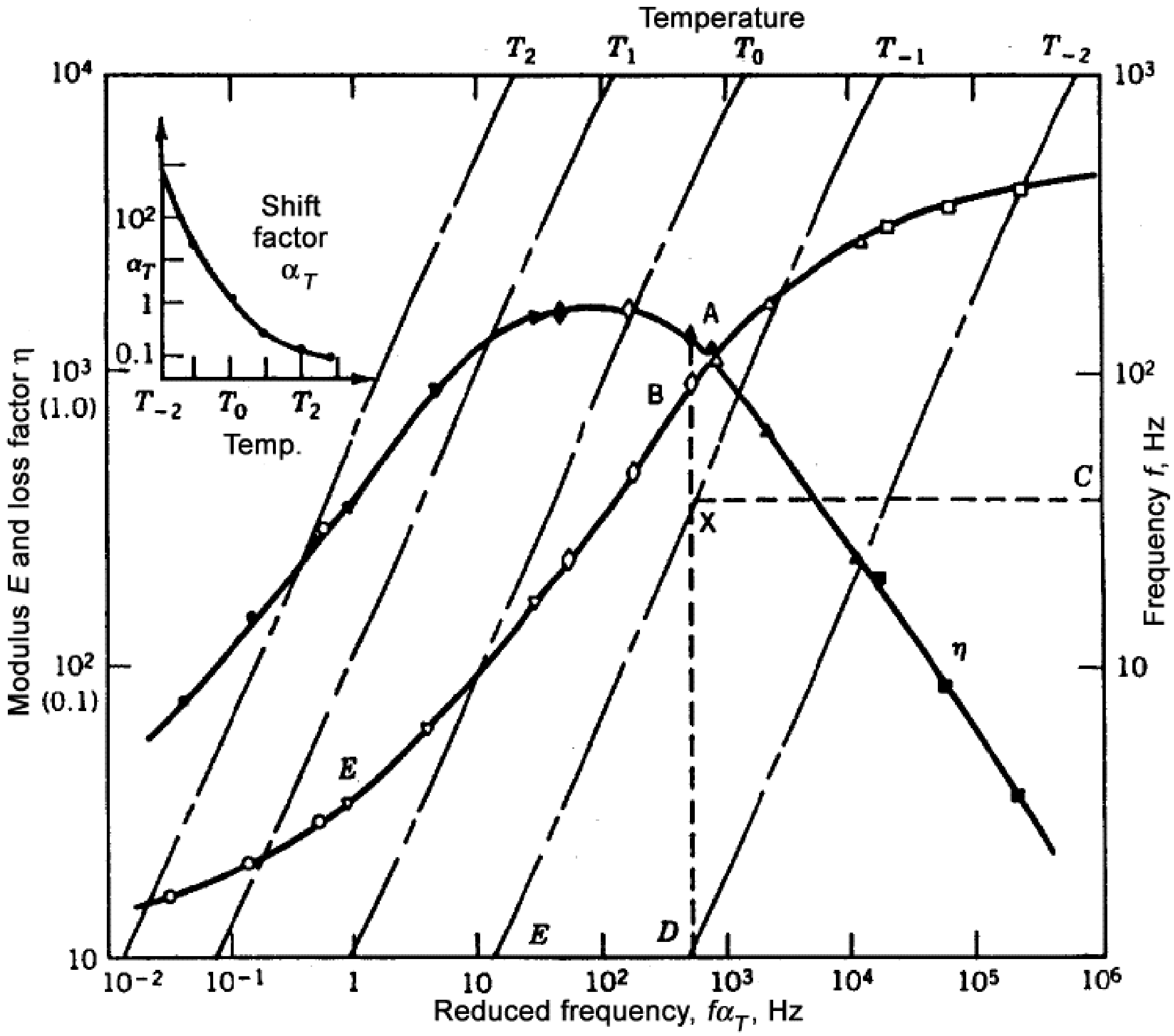


Variation of the storage modulus and loss factor with temperature.
Reference [9]



Frequency dependence of modulus and loss factor for various types of materials.

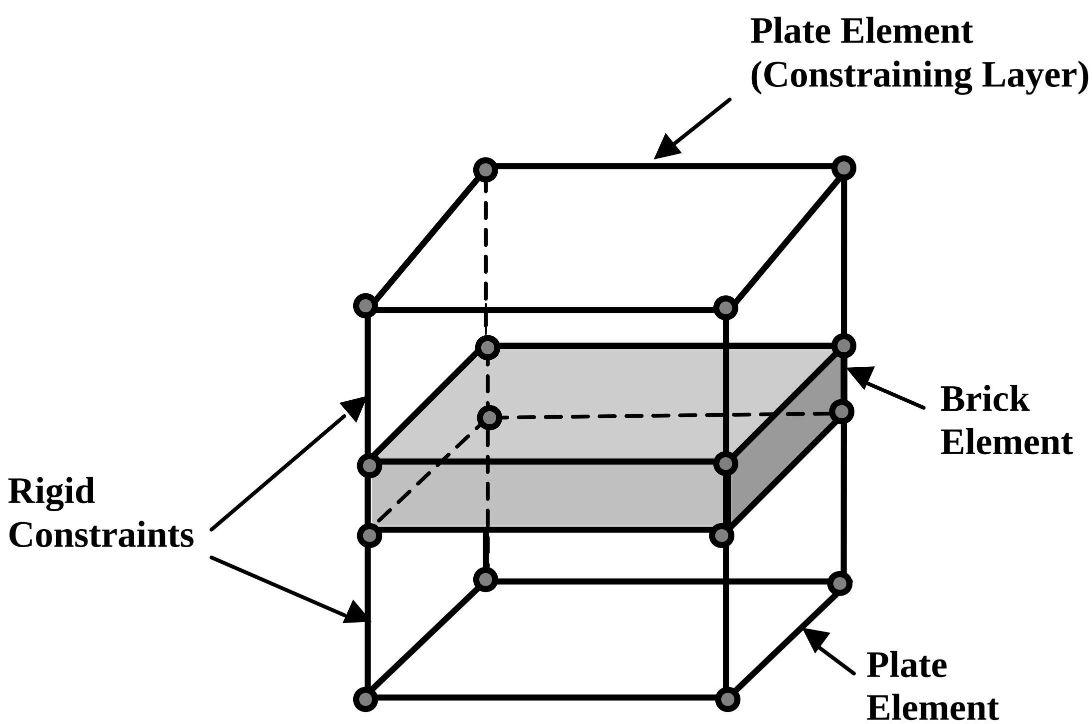
Reference [9]



Variation of E and h with reduced frequency fa_T.
Also construction of reduced temperature nomogram.

II) Techniques for Representing CLD Using the Finite Element Method (FEM)

a) Plate-Brick-Plate Elements



The model provides several advantages:

- **Accuracy**
- **No restriction with regard to the rigidity and thickness of the core and faces need be imposed.**
- **No limitation on the core thickness need be imposed.**
- **Model can be used with anisotropic layers.**

b) Plate-Beam-Plate Model

Disadvantages of Plate-Brick-Plate Elements Model

- **The FEM solution with solid brick elements to model the thin VEM layer requires a great deal of computation time.**

- **Though accurate, the cost may not justify the method**

The following is a much simpler model that also provides quite accurate solution:

- **The plate element is used, as in the previous case, to model the base and the constraining layer.**

- **The brick elements are replaced with beams connecting the base elements and the constraining layer elements.**

- **The structural damping for these beams are specified as:**

The spring variable in shear:

$$K_s^* = G^* A / L$$

The spring variable in extension:

$$K_e^* = E^* A / L$$

E^* = VEM's complex Young's modulus
G^* = VEM's complex shear modulus
A = cross sectional area of the beam determined by the modeling of the base and constrained layers (£1)
$L = H_2 + (H_1 + H_3) / 2$

<u>Note</u>

- **No need to use offset elements or rigid connections.**

- **In practice, it is convenient to select a beam of rectangular cross section and set A = 1. This would imply that the VEM layer carries all the uniformly distributed shear.**

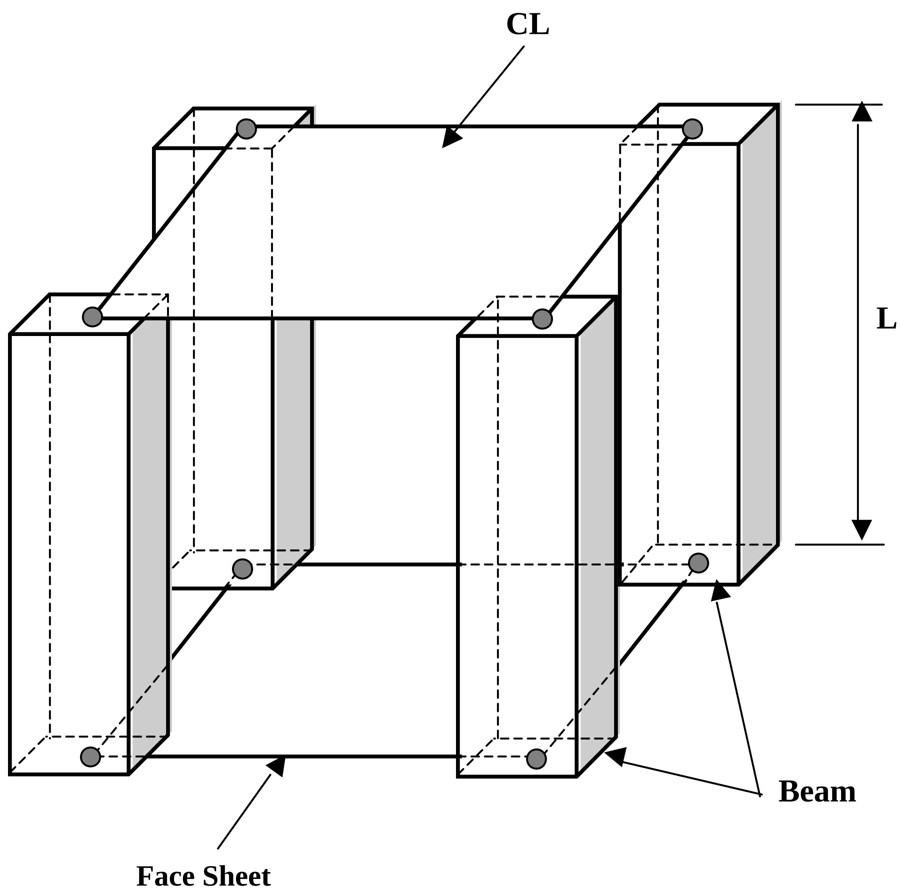
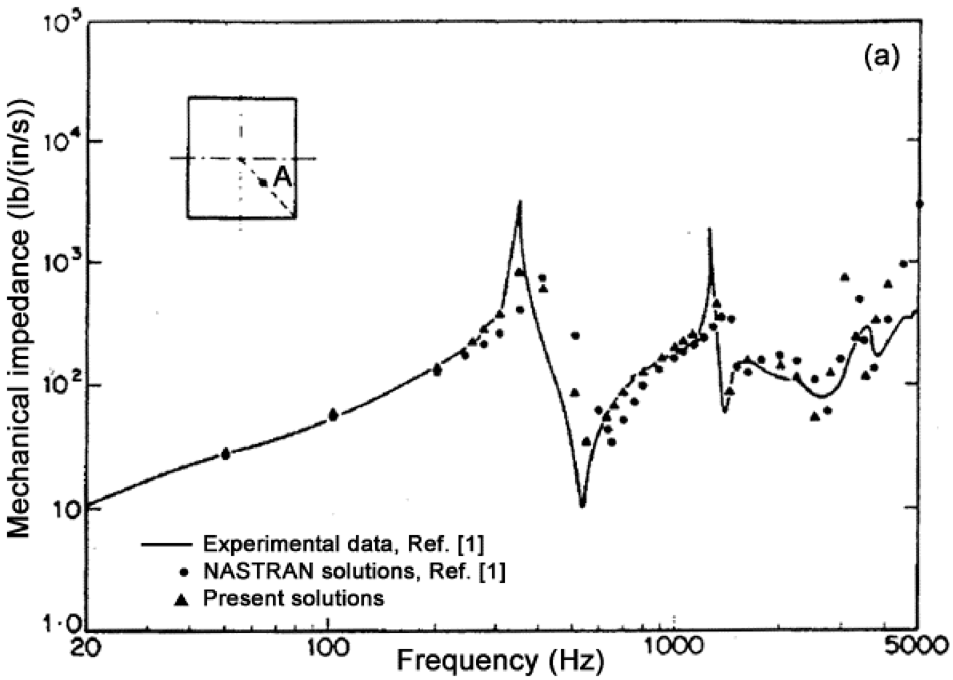
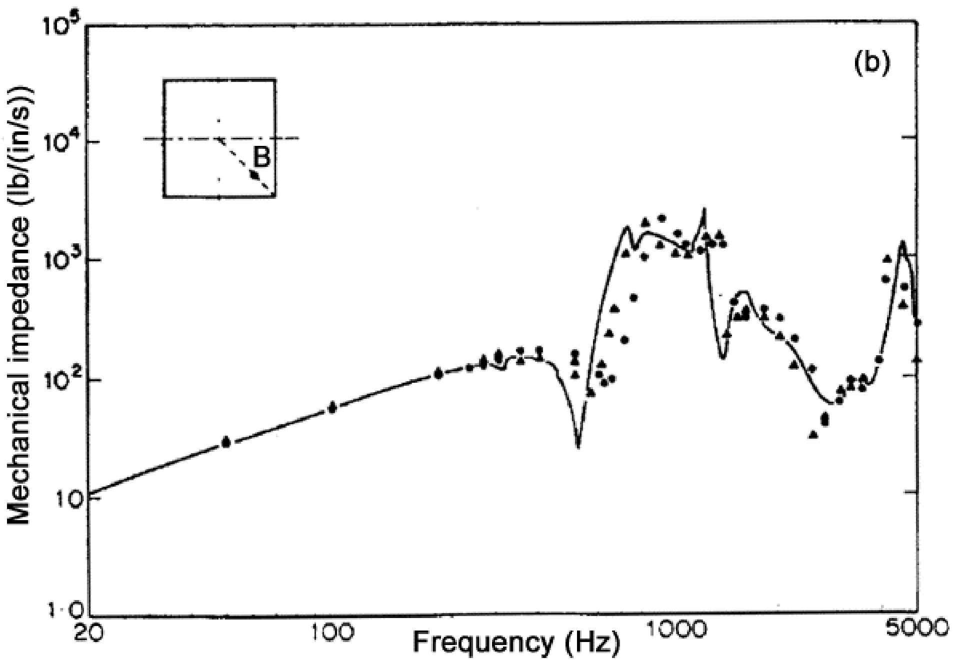


Plate-Beam-Plate Model

Plate-Beam-Plate Model





Transfer impedances of a damped plate:
Plot (a) at point A and plot (b) at point B.

III) Methods of Calculation

a) Direct Frequency Method

The NASTRAN's direct frequency response analysis is recommended as follows (the analysis is performed at a fixed temperature):

For a given frequency, calculate the core's shear modulus, G^*, the spring variables K^* and K_s^*, determine the complex stiffness matrix [K], and finally calculate the desired quantities.

Repeat this process for the range of frequencies of interest.

Note

UAI/NASTRAN supports direct frequency approach for viscoelastic materials.

Advantages of Direct Frequency Method

- **No assumption is made with respect to the level of damping.**

- **The frequency dependency of the materials of the structure's components can be accounted for if the stiffness matrix [K] is recalculated each time a new frequency is defined.**

Disadvantages of Direct Frequency Method

- **Very time consuming, especially if the [K] has to be recalculated for each frequency input.**

- **Depending on the geometry and level of damping, there may be coupling between modes, and therefore it may not be possible to excite one mode without exciting the other.**

b) Modal Method

- **In FEM discretization, the governing equations is**

$$\mathbf{[M]\{\ddot{U}\}} + \mathbf{[K]\{U\}} = \mathbf{\{F\}}$$

where [M] = global mass matrix, [K] = [K’] + i[K”] = global complex stiffness matrix, {U} = global complex displacement vector, and {Ü}= global complex acceleration vector.

- **Subjecting the structure to a harmonic excitation force {F}=** $\{\mathbf{F_o e^{iwt}}\}$**, then,**

$$\left(\mathbf{[K]} - w^{2}\mathbf{[M]}\right)\mathbf{\{U\}} = \mathbf{\{F_o\}}$$

Expressing {U} in terms of the eigenvectors of

$$\left(\mathbf{[K^{'}]} - w^{2}\mathbf{[M]}\right)\mathbf{\{U\}} = \mathbf{\{0\}}$$

we get

$$\mathbf{\{U\} = [}F\mathbf{]\{q(t\}}$$

$$\mathbf{\{\ddot{q}\}} + \mathbf{[}L + \mathbf{i\overline{H}]} = \mathbf{\{Q\}}$$

Note that this is expansion in an infinite space. In practice, however, one retains the first few terms of the expansion:

$$u_i(\tilde{x},t) = \sum_{j=1}^{N} \varphi_{i,j}(\tilde{x})\, q_j(t)$$

<u>Note</u> UAI/NASTRAN supports two other damping options, in addition to viscous and structural damping, with modal approach:

1) **Define the damping as a fraction of the critical damping to uncouple the equations.**

2) **Used only in transient response, define an equivalent viscous damping independently for each modal DOF. This method converts the structural damping from imaginary stiffness terms to real viscous terms.**

<u>Disadvantages of Modal Method</u>

- **Approximation within Approximation**
- **In the case of VEM, the resonance frequencies cannot be known, and therefore, the shape mode cannot be known.**

c) Dynamic Loading Considerations

- **Clearly, accurate modeling of the input load in the finite element method (FEM) is crucial.**

- **The power spectral density (PSD), due to its inherent definition, tends to smooth out the actual input. This may account for some of the minor discrepancies between test and computed results. See also the section on damping due to acoustic radiation.**

IV) Loss Factor Using CLD

- **The loss factor, h, of a constrained-layer damped structure depends on H_1, H_2, H_3, E_1, E_3, G, and h_2.**

- **Furthermore, because of G and h_2, it also depends on frequency and temperature.**

- **Let**

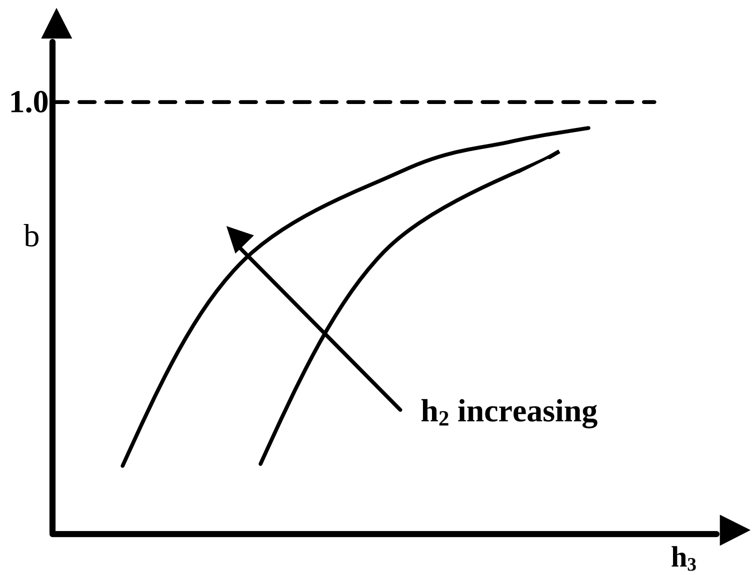
$$
\begin{aligned}
h_3 &= H_3/H_1 \\
h_2 &= H_2/H_1 \\
e &= E_3/E_1 \\
Y &= [3h_3(1 + h_3)] / (1 + h_3^3) \\
b &= h/h_2
\end{aligned}
$$

- **Discounting all other damping effects, the quantity $b < 1$.**

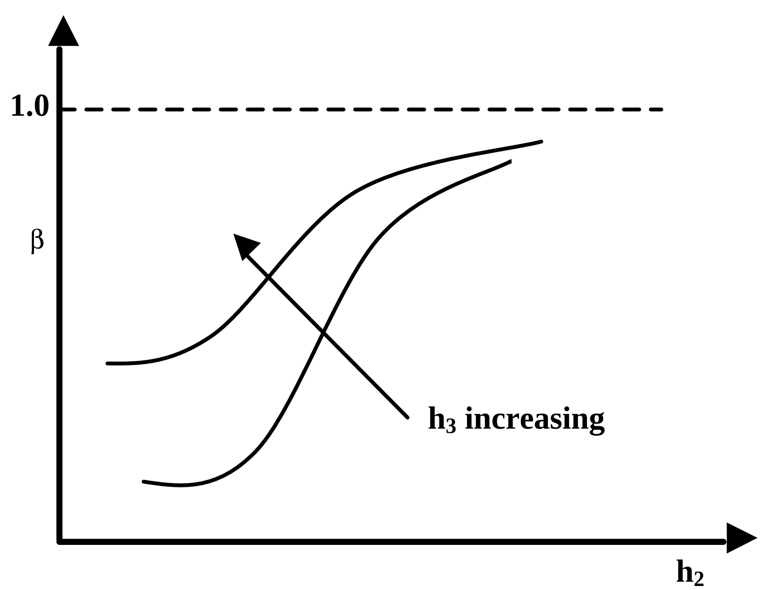
- **The value of $b_{max} = h_{max}/h_2$ approaches 1 as Y becomes larger.**

- **The loss factor h, in general, increases, up to a point, as h_3 increases.**

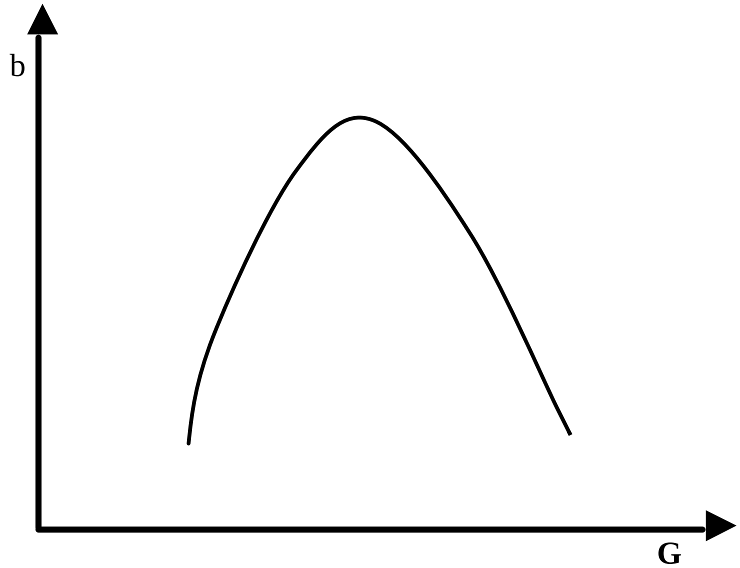
- **It decreases for all modes of vibration (those containing the additional rotary and translatory inertia terms) when the thickness of the core increases except for the flexural mode of vibration.**



Variation of b **vs h_3 in the case of flexural mode of vibration only**



. Variation of b **vs h_2 in the case of flexural mode of vibration only**



- **Maximum value increases and gets shifted to left as h_3 increases. The increase due to h_2, however, is considerably less than that due to h_3.**

- **Increasing the stiffness of the constraining layer, that is of e, will introduce more shear strain into VEM.**
- **Normally, however, e should not exceed 1.**
- **Therefore, the maximum amount of shear strain obtained in sandwich damping occurs when a VEM core is sandwiched between two layers of structures of same material and identical geometry.**
- **In situations where this is not possible, one may increase the stiffness of the constraining layer by additional constrained-layer treatment.**
- **The shear strain introduced into the second VEM layer is significantly less than that of the first layer. The effect of the additional constrained damping layers is primarily intended to increase the stiffness of the first constraining layer.**

- **For beams of rather simple geometry, it has been found, both experimentally and theoretically, that b is independent of h_2. This is, however, not the case for plates and other complex geometries.**

Evaluation of Loss Factor

- The evaluation of loss factor for a complex structure such as MAP S/C is very difficult and requires considerable care, and the technique used must be evaluated very carefully.

- There are, in general, two methods, direct frequency response and modal strain energy, used for evaluating the overall loss factor.

- For constrained layer damping, the modal analysis pose greater difficulty because of the lack of knowledge about accurate values of modal frequencies.

- On the other hand, the direct frequency response approach, though applicable naturally, is time consuming and for relatively highly damped systems may pose additional problems: coupling between modes, which means it might not be possible to excite one mode without exciting the other.

- In view of the difficulties involved in adopting either case, it is therefore recommended to use the iterative approach described below.

- The loss factor h of a structure executing steady state vibration, and one whose components' properties are frequency independent, can be defined in terms of energy quantities as

$$h = \frac{W_d}{2p\mathbf{W}} \qquad \textbf{(H1)}$$

  where W_d is the energy dissipation per cycle and W is the total strain energy stored in an identical undamped structure.

Techniques for Estimating the Loss Factor

As stated before, the two most commonly used approaches are: *direct frequency response method* and the *modal strain energy method*.

a) Direct Frequency Response Method

- **Actually no need to calculate the loss factor**

- **As an extra bonus, the loss factor can be calculated using the energy ratio method, Eq. (H1). After calculating the resonant frequency and the resonant displacement, $\hat{U} = \hat{U}' + i\hat{U}''$, the loss factor is calculated as follows**

$$h = \frac{\{\hat{U}^{*}\}[K''][\hat{U}\}}{\{\hat{U}^{*}\}^{T}[K'][\hat{U}\}} \qquad \textbf{(H2)}$$

where $(\ )^{*}$ is the complex conjugate of ( ).

<u>Advantages and Disadvantages</u>

As stated previously.

b) Modal Strain Energy Method

- **In the absence of frequency dependent materials, this is the preferred method. The loss factor of the r^{th} mode of the structure can be calculated from**

$$\eta^{(r)} = \frac{\mathbf{W}_{\mathbf{d}}^{(r)}}{\mathbf{W}^{(r)}}\,\eta_{\mathbf{d}} \qquad \textbf{(H5)}$$

where

$\eta^{(r)}$ **= the loss factor of the r^{th} mode of the structure**

$\eta_{\mathbf{d}}$ **= the loss factor of the VEM**

$\mathbf{W}^{(r)}/\mathbf{W}_{\mathbf{d}}^{(r)}$ **= the fraction of the elastic strain energy attributable to the VEM core when the structure deforms in its r^{th} mode shape**

- **In an FEM setting, Eq. (H5) leads to**

$$\eta^{(r)} = \eta_{\mathbf{d}} \frac{\sum_{e=1}^{n} \varphi_{\mathbf{e}}^{(r)} \mathbf{k}_{\mathbf{e}} \varphi_{\mathbf{e}}^{(r)}}{\varphi^{(r)T} \mathbf{k} \varphi^{(r)}} \qquad \textbf{(H6)}$$

where

$\varphi^{(r)}$ **= r^{th} shape mode**

$\varphi_{\mathbf{e}}^{(r)}$ **= subvector found by deleting from $\varphi^{(r)}$ all entries not corresponding to motions of nodes of the e^{th} viscoelastic element**

$\mathbf{k}_{\mathbf{e}}$ **= element stiffness matrix of the e^{th} viscoelastic element**

k = stiffness matrix of the entire composite structure

n = number of viscoelastic elements in the model

- **Notice that Eq. (H6) is <u>not</u> precisely true because it assumes that energy dissipated depends only on strain energies associated with the undamped mode shapes. Nevertheless, in cases where material properties are relatively independent of frequency, it produces satisfactory results. More on this later.**

Cases Where Material Properties Depend on Frequency

- **Clearly, the modal strain energy approach, as is, will produce incorrect results if the material properties of one, or more components, of the structure depend on the frequency.**
- **There are several ways to modify the approach to make it suitable for such situations; each having drawbacks and advantages.**

1. **Direct frequency approach**
2. **Correction factor**
3. **Iteration approach**
4. **Combination of modal strain and direct frequency approach**

1. Direct Frequency Approach

This method was discusses earlier.

2. Correction Factor

- **Some investigators have suggested to use the following form:**

$$\eta^{(r)} = \left(\frac{\Omega_d^{(r)}}{\Omega^{(r)}} \eta_d \right) f(\omega) \qquad \text{(H7)}$$

where $f(\omega)$ is a suitable function of frequency; the remaining terms are as defined in Eq. (H6).

- **The form of $f(\omega)$ that would produce accurate results, however, requires extensive testing and curve fitting.**
- **One suggested formulation, that has been adopted for MAP's analysis, is**

$$\eta^{(r)} = \left(\frac{W_d^{(r)}}{W^{(r)}} \eta_d \right) \sqrt{\frac{G_d\left(\omega^{(r)}\right)}{G_{d,ref}}} \qquad \textbf{(H8)}$$

where

$G_d\left(\omega^{(r)}\right)$ = VEM's shear modulus at rth mode frequency

$G_{d,ref}$ = VEM's shear modulus used in the final normal modes calculation to obtain modal frequencies, shapes, and masses

- **The formulation is certainly an improvement, but nevertheless an approximation.**
- **It is inadequate for complex geometries such as MAP.**
- **Test results shows considerable discrepancies, even for a simple geometry, between analytical solution, using differential equations, and FEM results.**
- **To be of any value, the expression must be evaluated for every given frequency, which, if implemented, essentially requires direct frequency method.**

3. Iteration Method

Procedure to Determine the Eigenvalues

For a given temperature,

1. **Determine G^* as a function of w.**

2. **Assume an initial (starting) w^{i}, $i = 0$, and calculate G^* and [K].**

3. **Calculate the eigenvalues $w_1^{i}, w_2^{i}, w_3^{i}, \ldots$**

4. **For each j in w_j^{i} do the following until certain convergence criteria is met.**

   4.1 Determine [K] and a new set of eigenvalues $w_1^{i+1}, w_2^{i+1}, w_3^{i+1}, \ldots$

   4.2 Set i -> i+1

The process is expected to converge after a few iterations.

4. Combination of Modal Strain and Direct Frequency Approaches

- **Locate the resonant frequency first using the direct frequency response.**

- **The deflected shape of the damped structure is then used to calculate the strain energy fractions stored in the element.**

- **In this method, the complex eigenvalue problem is not solved.**

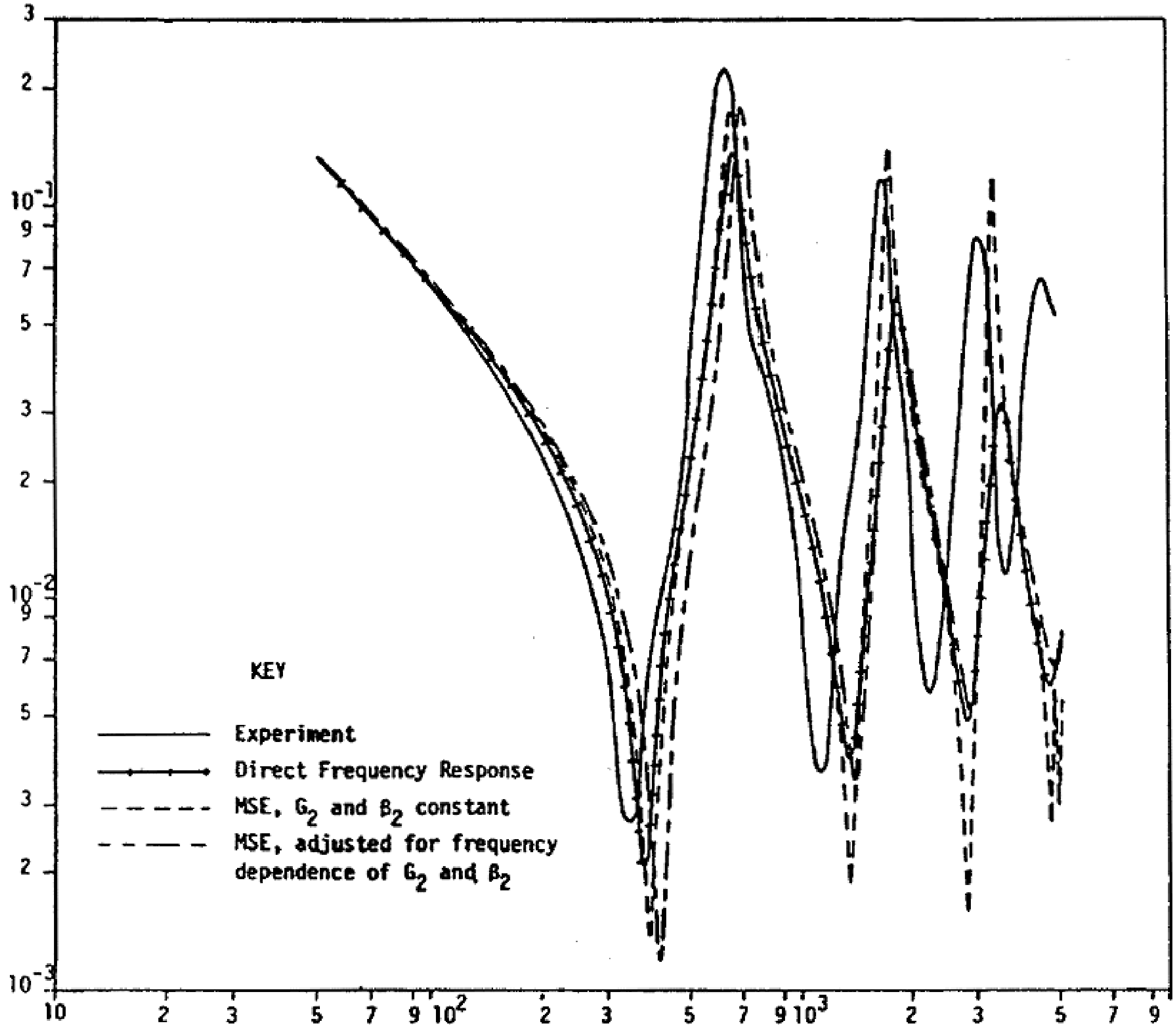
- **Instead, the damped resonance frequencies and corresponding modal damping factors are obtained for a number of frequencies.**

- **The loss factors of the overall structure, η_i, are obtained as the weighted sum of the loss factors of the individual elements:**

$$\eta_i = \frac{\sum_{e}^{n} \eta_e W_{j,max}^{(i)}}{\sum_{e}^{n} W_{j,max}^{(i)}} \quad \textbf{(No Need; A Bonus)}$$

where n = number of elements, $W_{j,max}^{(i)}$ = the peak elastic strain of element j in the i^{th} mode. For better accuracy, the damped deflection curves, rather than the undamped normal modes, are used.

- **Clearly, in this approach the computational benefits of the modal strain energy are lost.**



Driving point velocity admittance of sandwich ring.

V) Load Path Assumptions

- **If there is a load path between the constraining layer (CL) and the base plate, the CL carries an axial load F_3. The static longitudinal equilibrium then requires that $F_1 + F_3 = 0$.**
- **This results in**

$$E_1A_1u_1 + E_3A_3u_3 = 0 \qquad (*)$$

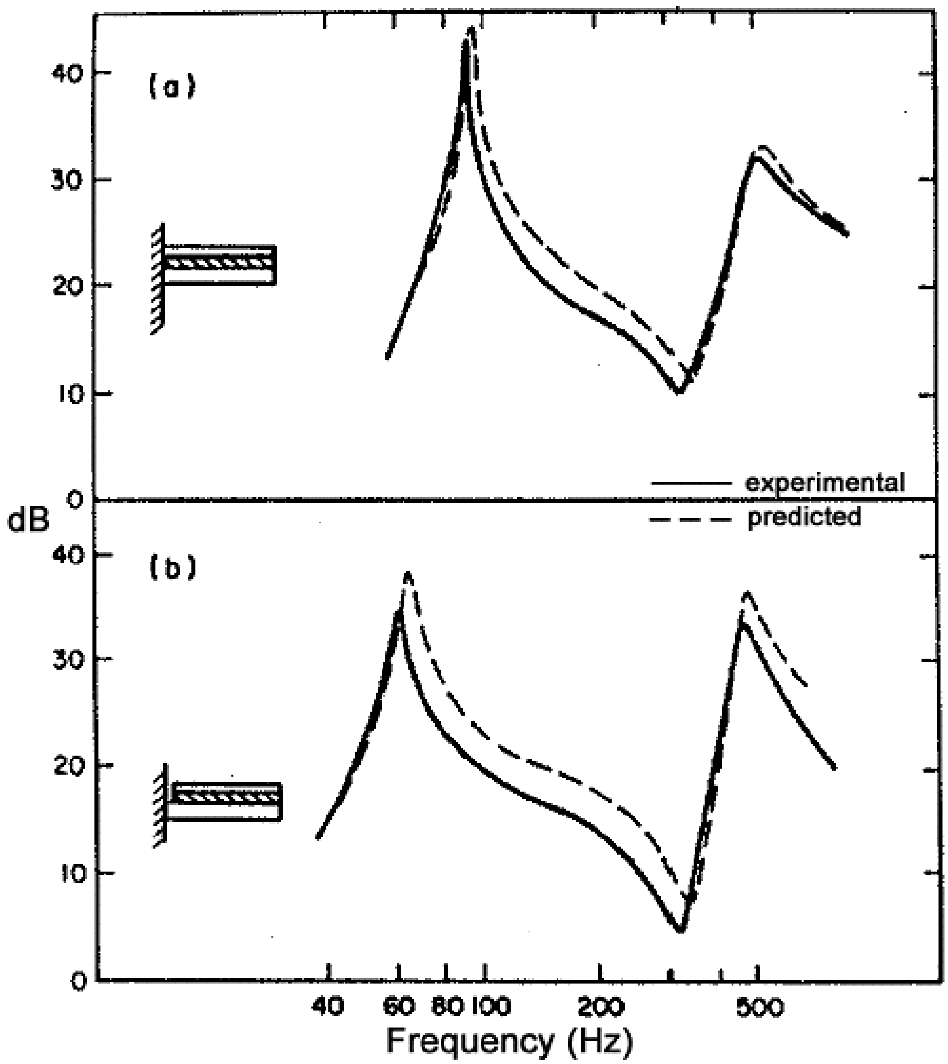
or

$$u_1 \propto u_3 \qquad (**)$$

- **Now, $u_3 = 0$ implies $u_1 = 0$ and vice versa. In some cases this may be inconsistent with the actual damping treatment.**

- **For example, when the plate is clamped or fixed at the boundary (clamped-free), while CL is free-free.**

- **Relaxation of relation (*) will introduce four additional degrees of freedom (two in the case of a beam): the in-plane displacements u_3 and v_3, and two rotations ϕ_3 and ψ_3.**



(a) Constraining elastic layer clamped-free;

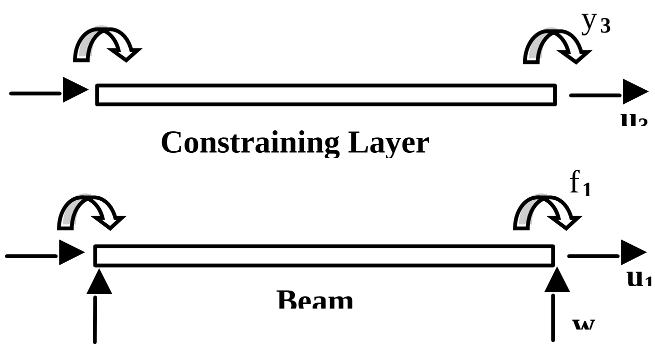
Ordinate is 20log|g|***F*****|** (γ and *F* are the acceleration and external force at the end of the beam, respectively)

Abscissa is frequency

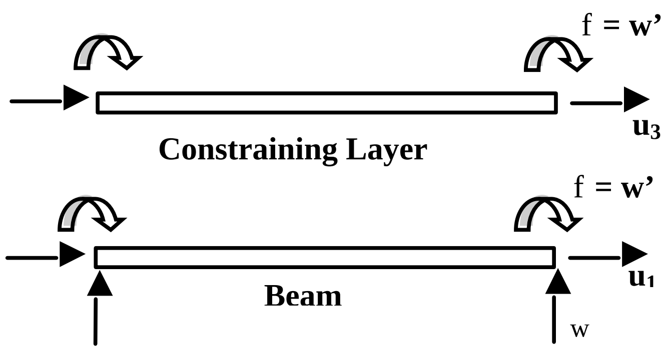
(b) Constraining elastic layer free-free;

(legend same as (a))

For a deep beam with no load path:



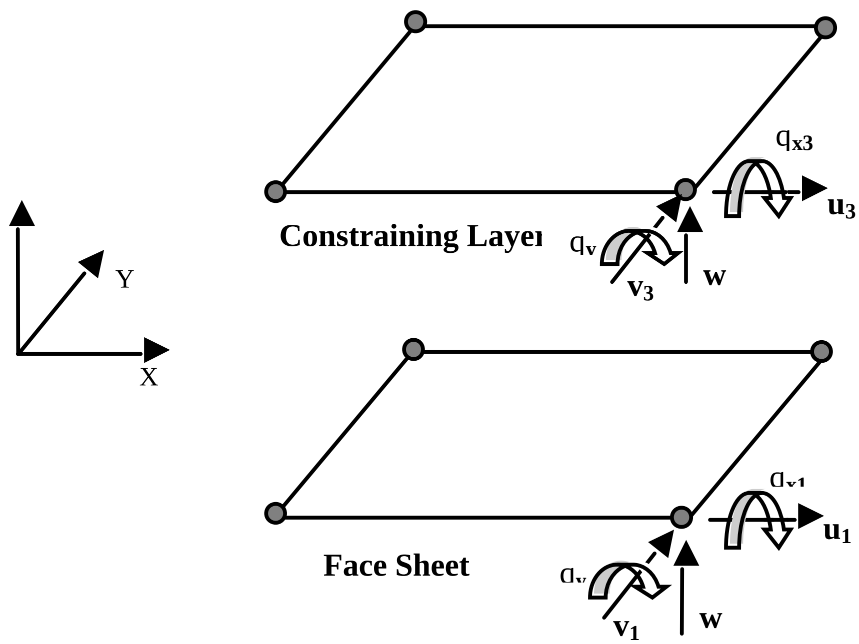
For narrow beam with no load path:



Thick Plates – No Load Path

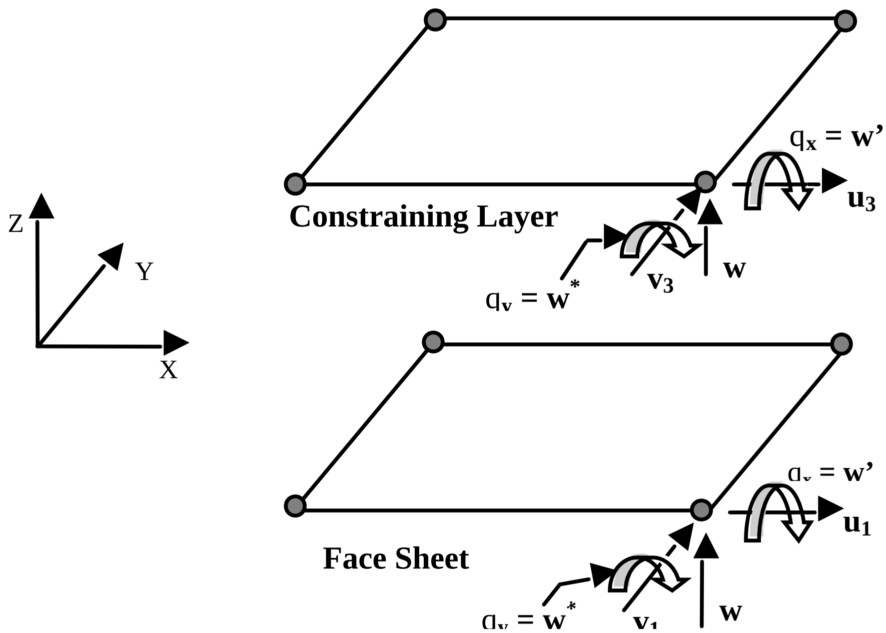
- **When the wavelengths are less ten times the plate thickness, shear deformations and rotary inertia effects must be included in the analysis.**

- **As a result, when shear deformation is important, it cannot be assumed that normal to the middle surface of the plate remain normal during deformation.**



Thin Plates – No Load Path

- It is assumed that normals to the middle surface of the undeformed plate remain straight and normal to it during deformation.
- Furthermore, it is assumed that the stress in the transverse direction, σ_z, is zero.



- Here, $w' = \partial w/\partial x$ and $w^* = \partial w/\partial y$.

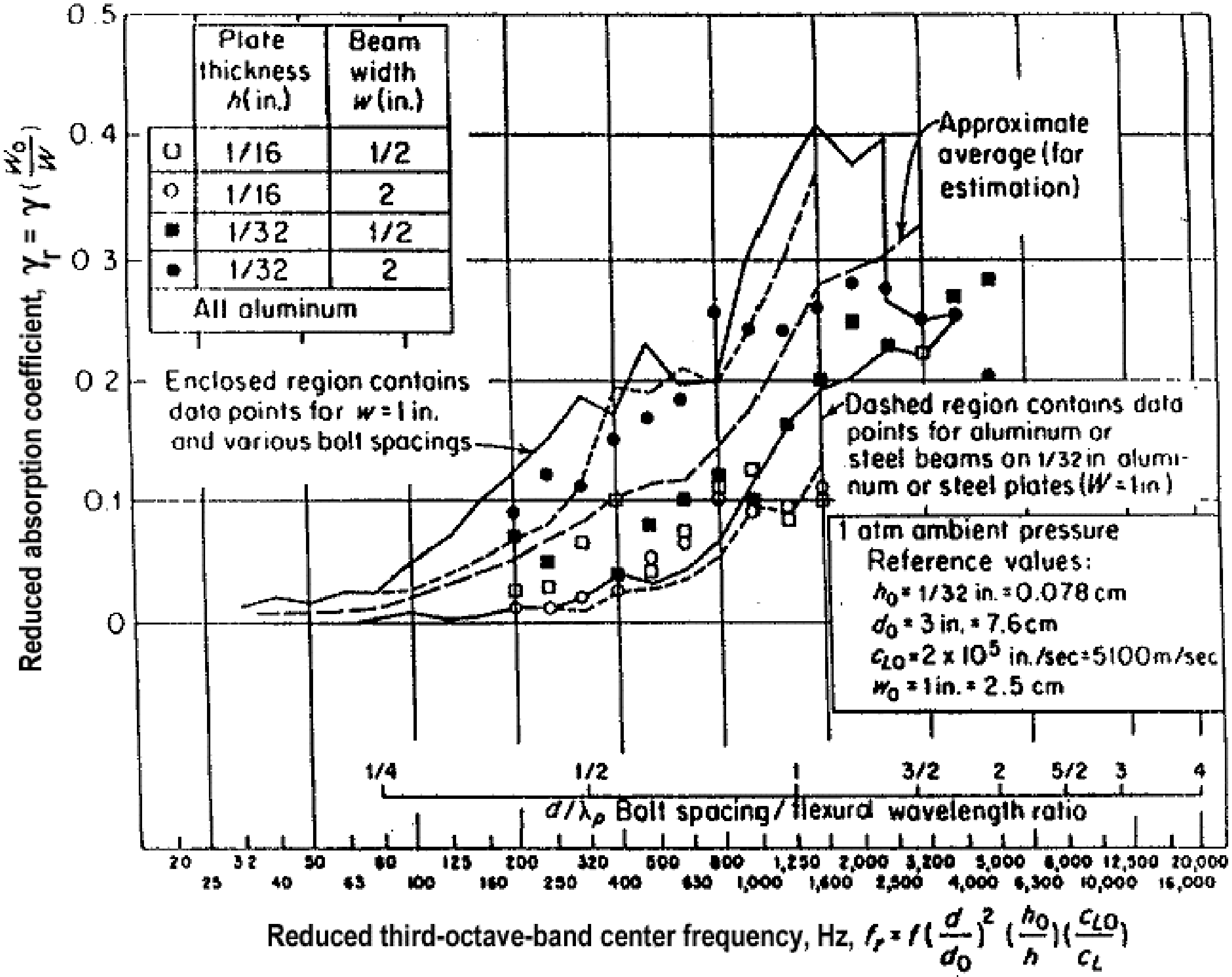
- In both cases it is assumed that the transverse displacement (bending), w, is the same in all layers.

VI) Other Damping Considerations

a) Damping of Bolted Joints

b) Struts

c) Composite/Honeycomb Panels



Summary of reduced absorption coefficient data for beams fastened to plates by rows of rivets, bolts, or spot welds. Ref. [17]

VII) Effects of Inertia Terms

- **The basic equations of bending vibration of unsymmetrical viscoelastically damped constrained-layer sandwich plates employing dissimilar elastic layers should, in general, include the following effects:**

  e1. Effects of flexural and membrane energies in the faces
  e2. Effects of transverse shear in the core
  e3. Effects of transverse inertias in both core and faces
  e4. Effects of rotary and translatory inertias in both core and faces

- **In a majority of both experimental and analytical investigations, the last effects, e4, have been neglected.**

- **The assumption is valid at low frequencies where transverse inertia is predominant.**

- **At high frequencies these effects are, however, of quite importance and their neglect may lead to erroneous results.**

- **In fact, in the case of sandwich plates they are of considerable importance at even relatively lower frequencies.**

- **The transverse response is relatively unaffected by the inclusion of the rotary and translatory inertia terms.**

- **The longitudinal responses of both face and constraining layer, however, are considerably affected.**

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